Директору Математичког института САНУ
Проф. др Зорану Огњановићу
Научном већу Математичког института САНУ
Академику Проф. др Драгошу Цветковићу

Стручни извештај о учешћу у међународној научној конференцији

The 10th International Workshop
Computer Algebra Systems in Teaching and Research (CASTR'2019)
Siedlce, Poland, September 25 – 29, 2019.
a mini-symposium on Celestial and Classical Mechanics in the framework of the confeence CASTR

ORGANIZERS:
Siedlce University of Natural Sciences and Humanities (Siedlce, Poland)
Warsaw University of Life Sciences - SGGW (Warsaw, Poland)
website: http://www.castr.uph.edu.pl

Поштовани директоре, Проф. Огњановићу,
Поштовани председниче, Проф. Цветковићу,
Цењени чланови Научног Већа МИ САНУ,

У периоду од 24-29.09.2019. године учествовала сам у раду међународне научне конференције: The 10th International Workshop Computer Algebra Systems in Teaching and Research (CASTR'2019), Siedlce, Poland, September 25 – 29, 2019, у организацији Siedlce University of Natural Sciences and Humanities (Siedlce, Poland) и Warsaw University of Life Sciences - SGGW (Warsaw, Poland).

Конференција је одржана на Siedlce University of Natural Sciences and Humanities (Siedlce, Poland).

Моје учешће се састојало у чланству у Научном комитету **CASTR'2019,** и организацији мини-симпозијума: **Mini-symposium on Computational Aspects of Classical and Celestial Mechanics, Stability and Motion Control,** који је планиран и реализован у оквиру научног програма **CASTR'2019 (in the framework of the workshop).**

Традиционално, организатори овог Мини-симпозијума су:

Pavel S. Krasilnikov (Moscow, Russia),
Katica R. (Stevanovic) Hedrih (Belgrade, Serbia),
Alexei A. Tikhonov (St. Petersburg, Russia)

Моје учешће се огледало и у одежавању два везана пленарна предавања под називима:

Acceleration of the second order (Jerk) of a kinetic point moving with constant sectorial velocity as well as of the planet moving

Paper dedicated to the memory of my professor of Mechanics,
Professor Dr.Ing. Dipl. Math. Danilo P. Rašković (1910-1985)

Acceleration of the second order – jerk of a rigid body rotates around a fixed point

Paper dedicated to memory of my professor of Mechanics,
Professor Dr.Ing. Dipl. Math. Danilo P. Rašković (1910-1985)
and half century from first common publication

Ова пленарна предавања су била прва у програму Минисимпозијума, који је реализован другог и трећег дана ове **CASTR'2019** конференције. Први дан конференције је био посвећен области Computer Algebra Systems in Teaching and Research. У оквиру Минисимпозијума, био је већи број квалитетних радова и саопштења из класичне, а посебно из небеске механике и динамике небеских тела.

Највћи број учесника ове Конференције је био из Пољске, али је било учесника и из иностранства, Немачке, Русике, Србије, Казахстана, Украјине и др.

Организатори су, по традицији, публиковали један том публикације под нативом:
Computer Algebra System in Teaching and Research (CASTR), у издању Sedlice University of Natural Sciences and Humanities, Institute of Mathematics and Physics, Vol. VIII, ISBN 978-83-7051-956-8 .

У тој публикацији публикована су моја два рада, чији садржај обухвата садржаје пленарних предавања. Напомињем да су пре 53 године ови садржаји о убрзању другог реда публиковани на сропском језику, у публикацијама Универзитета у Нишу и Машинског факултета у Нишу, као мји први публиковани научни радови. Како је садржај истих актуелан и данас, и за универзитетску наставу и за истраживање, ја сам те садржаје, проширила, допунила и превела на енглески језик, и како су добили позитивне рецензије компетентних рецензената, а прихваћени од уредника публикације, то су публиковани у истој, и посвећени мом професору механике др Данилу П. Рашковићу, који да је данас жив, би се радовао њиховом публиковању. На радовима су назнаке и цитирања ка изворним референцама. Библиограгски подаци тих радова су:

Katica R. (Stevanović) Hedrih, (2019), Acceleration of the second order – jerk of a rigid body rotates around a fixed point, Paper dedicated to memory of my professor of Mechanics, Professor Dr. Ing. Dipl. Math. Danilo P. Rašković (1910-1985) and half century from first common publication, CASTR Sedlice 2019, Computer Algebra system inTeaching and Research, Sedlice University of Natural Sciences and Humanities, Institute of Mathematics and Physics, Vol. VIII, pp. 184-210, ISBN 978-83-7051-956-8

Katica R. (Stevanović) Hedrih, (2019), Acceleration of the second order (Jerk) of a kinetic point moving with constant sectorial velocity as well as of the planet moving, Paper dedicated to the memory of my professor of Mechanics, Professor Dr.Ing. Dipl. Math. Danilo P. Rašković (1910-1985), CASTR Sedlice 2019, Computer Algebra system inTeaching and Research, Sedlice University of Natural Sciences and Humanities, Institute of Mathematics and Physics, Vol. VIII, pp. 211-231, ISBN 978-83-7051-956-8

У прилогу Извештаја достављам: серију докумената из којих се закључује сва озбиљност научне селекције и највиши научни ниво научног програма ове интернационалне конференције

The 10th International Workshop
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не много обимне по броју учесника, али веома високог нивоа по приказаним научним резултатима из обе области Компјутерских алгебарских система у настави и истраживању, као и у класичној и небеској механици.

У Београду, 19. октобар 2019. године

Професор др Катица (Стевановић) Хедрих

Mini-symposium on Computational Aspects of Classical and Celestial Mechanics, Stability and Motion Control is planned to be organized in the framework of the workshop.

Organizers:
Pavel S. Krasilnikov (Moscow, Russia),
Katica R. (Stevanovic) Hedrih (Belgrade, Serbia),
Alexei A. Tikhonov (St. Petersburg, Russia)

Scope

- Computer Algebra Methods for solving ODEs and PDEs
- Modelling of the Galactic Cosmic Rays Propagation
- Computer methods in Classical and Celestial Mechanics
- Nonlinear Optimization Problems
- Computer Algebra Methods in Finance and Economics
- Computer Algebra Systems in Education
- Symbolic-Numeric Computations

Scientific Committee

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OF NATURAL SCIENCES AND HUMANITIES

Vice-Rector for Research

Mirosław Minkina
Professor

Konarskiego 2,08-110 Siedlce tel:25 643 19 42,e-mail reknauka@uph.edu.pl; www.uph.siedlce.pl

Siedlce, April 05, 2019

Professor Dr
Katica R. (Stevanović) Hedrih
Department of Mechanics
Mathematical Institute SANU
Knez Mihailova st. 36/III
Belgrade, 11009 Serbia
khedrih@sbb.rs

Dear Professor Dr Katica (Stevanović) Hedrih,

On behalf of the Organizing Committee of the 10th International Workshop on Computer Algebra Systems in Teaching and Research (CASTR'2019) to be held in Siedlce, Poland from September 25 till September 29, 2019, we are glad to invite you to participate in the conference and to present your talks:

1. Acceleration of the second order (Jerk) of a kinetic point moving with constant sectorial velocity as well as of the planet moving
2. Acceleration of the second order – jerk of a rigid body rotates around a fixed point

As an invited speaker you'll have 45 min. for presentation. We also expect that you'll agree to act as a Chairman at the mini-symposium on Classical and Celestial Mechanics.

Please note that the registration fee is 500 PLN. It covers organization expenses, conference materials and refreshment room at the Workshop. It can be paid on site upon arrival. Participants of the workshop CASTR'2019 will be accommodated at the UPH hotel situated in a walking distance from the Faculty of Science building, Zytnia str. 17/19, Siedlce, (http://www.uph.edu.pl/sprawy-studenckie/domy-studenckie).

We kindly ask you to confirm your participation by September 15, 2019 via e-mail: castr@uph.edu.pl and to let us know some details concerning your arrival and departure (number of train or flight, date and time). All the details concerning the workshop you can find on the website http://www.castr.uph.edu.pl

We look forward to seeing you in Siedlce.

Best wishes,

PROREKTOR DS. NAUKI
dr Mirosław Minkina
profesor nadzwyczajny

UNIWERSYTET
PRZYRODNICZO-HUMANISTYCZNY W SIEDLCACH

WYDZIAŁ NAUK ŚCISŁYCH

08-110 Siedlce, ul. 3 Maja 54, tel. +48 25 643 1079, e-mail: castr@uph.edu.pl

Siedlce, September 25-29, 2019

2019 C@STR
Computer Algebra Systems in Teaching and Research

Certificate of Attendance

This certificate is presented to

Katica R. Hedrih

Department of Mechanics,
Mathematical Institute of the Serbian Academy of Science and Arts,
Belgrade, Serbia

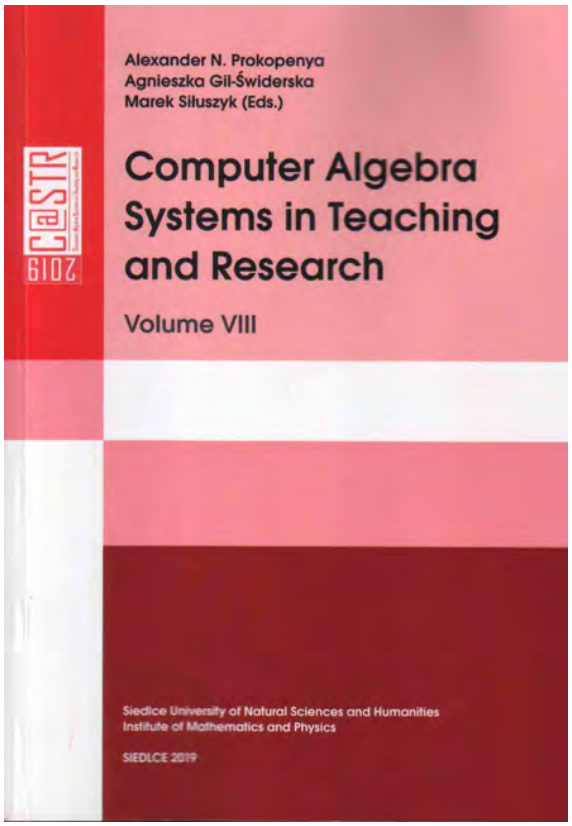
For attendance at

CASTR'2019, 10th International Workshop on Computer Algebra Systems in Teaching and Research (www.castr.uph.edu.pl)

Held in Siedlce, Poland, September 25-29, 2019.

Dr. Marek Siłuszyk

Organizing Committee of CASTR'2019
University of Natural Sciences and Humanities in Siedlce
3 Maja str. 54, 08-110 Siedlce, Poland
Tel. +48 25 6431076
http://www.castr.uph.edu.pl



ISBN 978-83-7051-956-8

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tel. +48 25 643 1003, e-mail: imif@uph.edu.pl

Contents

II. Problems of Classical and Celestial Mechanics

Acceleration of the Second Order – Jerk of a Rigid Body Rotates around a Fixed Point*

Paper dedicated to memory of my professor of Mechanics, Professor Dr. Ing. Dipl. Math. Danilo P. Rašković (1910-1985) and half century from first common publication

Katica R. (Stevanović) Hedrih*

**Department of Mechanics, Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, Serbia: E-mail: katicahf@mi.sanu.ac.rs; khedrih@eunet.rs; and Faculty of Mechanical Engineering, University of Nis, Nis, Serbia: E-mail: khedrih@ptt.rs*

Abstract: The results of the research on acceleration of the second order (a joke or a twitch) contained in two papers [3,4], published, in Serbian, half a century ago, are still presented, which are still current. The first paper is entitled:

"Acceleration of the second order (jerk or jark) of a material point that moves at a constant sectoral velocity."

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"Acceleration of the second order (twitch) when the body rotates around a fixed point".

The first part of this second work was shown using the matrix notation for description of the kinematics of motion, and authored by my brilliant Professor dr. Ing. Dipl. Math. Danilo Rašković (1910-1985). He was one of the head of the Department of Mechanics at Mathematical Institute of the Serbian academy of sciences and arts, and author of 150.000 examples of different books in area of theoretical and applied mechanics, published in Serbian language, and used by numerous generations of students of technical sciences. The second part of this work is shown by the vector notation and represents the author's scientific results. And the entire work belongs to the classical field of the kinematics of the body rotation around a fixed point.

This paper is expanded and amended work, first published in the Serbian language as: Rašković P. Danilo and Stevanović R. Katica *(later merried family name Hedrih), Ubrzanje drugog reda (trzaj ili džerk) krutog tela pri obrtanju oko nepomične tačke (Acceleration of second order of a rigid body rotates around fixed point), Zbornik radova Tehničkog fakulteta Univerziteta u Nišu. 1966/1967.* Also, it is very actual in present days.

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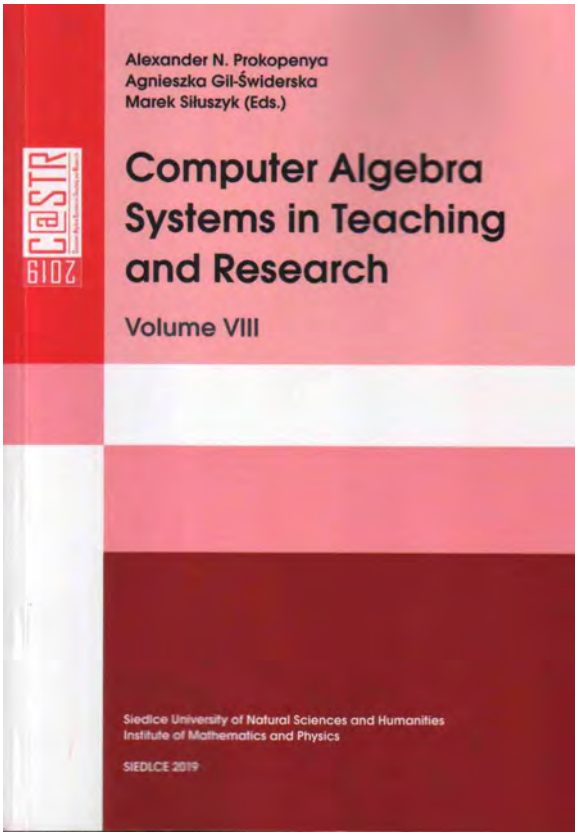

Contents

Acceleration of the Second Order (Jerk) of a Kinetic Point Moving with Constant Sectorial Velocity as well as of the Planet Moving*

Paper dedicated to memory of my professor of Mechanics, Professor Dr. Ing. Dipl. Math. Danilo P. Rašković (1910-1985) and half century from first common publication

Katica R. (Stevanović) Hedrih*

Department of Mechanics, Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, Serbia; E-mail: katicah@mi.sanu.ac.rs; khedrih@eunet.rs; and Faculty of Mechanical Engineering, University of Niš, Niš, Serbia; E-mail: khedrih@gmail.com

Abstract: The results of the research on acceleration of the second order (a jerk or jerking) of the mass particle moving with constant sector velocity in the plane are presented, in a published paper more than a half a century ago but in Serbian language.

This paper is titled:

"*Acceleration of the second order (or jerk or jerking) of a material point that moves with a constant sector velocity.*"

The first part of the article refers to the movement of the kinematic point along the path line, the change in velocity, and the acceleration of the point, when it is possible to talk about the change of the acceleration accelerator or the acceleration of the second order – jerk or jerking (or Ruch, Rucken, Jerk, pulse). The components of the vector of acceleration of the second order of the motion of a kinetic point in space are in all three directions; of the tangent, of the normal and of the binormal direction of the path line, in the general case of motion of the kinematic point in the space will also be displayed.

Then the components of the kinematic point of the vector of the acceleration of the second order of the motion of a kinetic point in plane, under the action of central force, and with constant sector velocity are presented.

Binet's formula is also used for necessary transformation in expressions of the components of the vector of acceleration of the second kind (Jacques Philippe Marie Binet (1786-1856)). A theorem on the relation between the circular component of the second-order acceleration vector of the kinematic point that moves with a constant sector velocity and the radial components of its acceleration vector are defined and proofed.

For examples of the central motion of the kinematic point, in a plane, by constant sector velocity, the expressions for the angular acceleration of the second order and the components of the vector of the acceleration second order, along the

* Expanded and amended work, first published in the Serbian language as: Stevanović R. Katica (*later married family name Hedrih*), *Ubrzanje drugog reda (trzaj ili džerk) materijalne tačke koja se kreće konstantnom sektorskom brzinom (Acceleration of second order of a material particle moving with constant sectorial velocity .* Naučni podmladak, 1967, str. 69-70.

Прилог 3.

Acceleration of the second order (Jerk) of a kinetic point moving with constant sectorial velocity as well as of the planet moving*

Paper dedicated to the memory of my professor of Mechanics,

Professor Dr.Ing. Dipl. Math. Danilo P. Rašković (1910-1985)

Katica R. (Stevanović) Hedrih[*],

[*]*Department of Mechanics, Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, Serbia; E-mail: katicah@mi.sanu.ac.rs; khedrih@eunet.rs; and Faculty of Mechanical Engineering, University of Nis, Nis, Serbia; E-mail: khedrih@sbb.rs*

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Then, the components of the kinematic point of the vector of the acceleration of the second order of the motion of a kinetic point in plane, under the action of central force, and with constant sector velocity are presented.

Binet's formula is, also, used for necessary transformation in expressions of the components of the vector of acceleration of the second kind (Jacques Philippe Marie Binet (1786-1856)). A theorem on

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the relation between the circular component of the second-order acceleration vector of the kinematic point that moves with a constant sector velocity and the radial component of its acceleration vector are defined and proofed.

For examples of the central motion of the kinematic point, in a plane, by constant sector velocity, the expressions for the angular acceleration of the second order and the components of the vector of the acceleration second order, along the Archimedes spiral, along the sinusoidal spirals and ellipses, are derived. These results are new in comparison with content of the source paper published half a century before.

On the basis of previous results, as well as Kepler's third law, the angular acceleration of the second order and acceleration of the second order of the planets, which perform central movement along elliptical paths and which are moving with constant sector velocity, are shown.

Key words: Acceleration of the second order; kinematic point; constant sector velocity; ellipse; Planet.

Прилог 4.

Acceleration of the second order – jerk of a rigid body rotates around a fixed point*

Paper dedicated to memory of my professor of Mechanics,
Professor Dr.Ing. Dipl. Math. Danilo P. Rašković (1910-1985)
and half century from first common publication

Katica R. (Stevanović) Hedrih[*],
[*]*Department of Mechanics, Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, Serbia; E-mail: katicah@mi.sanu.ac.rs; khedrih@eunet.rs; and Faculty of Mechanical Engineering, University of Nis, Nis, Serbia; E-mail: khedrih@sbb.rs*

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Acceleration of the second order – jerk (Ruch, Rucken, Jerk, pulse)(Word forms: jerks, jerking, jerked) plays an increasing role in various domains of mechanics and physics. The aim of this paper is to determine this acceleration of the second order, or shorter *jerk*, when a rigid body rotates around a fixed point. The corresponding relations for thee instantaneous angular velocities, angular accelerations, velocities, acceleration of the second order and jerk are expressed in matrix form and referred to the fixed point and instantaneous axes. The Rivals theorem is enlarged on the jerk also, and the natural components of the acceleration of the second order - jerk vector of a rigid body rotation around a fixed point are given. The case of the regular progressive precession is treated, also.

In concluding remarks, the contents of the original and new results, will be displayed on the topic of the dynamics of rotating the rigid body around a fixed point using the vectors of mass moments that are related to the pole and axis, introduced by the author in 1992. From this area, in special cases of rigid body dynamics when rotating around a fixed point, the most famous works and solutions are: Euler's solution, Lagrange's solution and the solution of Sofia Kovaljevskya. The general solution of the dynamics of the rigid body differential equations when rotates around a fixed point has not been found to this date.

About twenty years ago, the lecturer attended the Conference in Donetsk, organized by the Institute of Mechanics NANU from Donetsk, one *Round Table* and a discussion on one book and one paper, by two independent researchers, who individually claimed to have found a general solution a system of differential equations of the dynamics of a rigid body around a fixed point. It was a real "scientific octave" between the authors and the opponent’s important world scientist. The paper will make an attempt to display this discussion in Appendix of this paper..

Key words: Acceleration of the second order, jerk, rigid body that rotates around a fixed point, instantaneous axis, angular velocity, matrix method, vector method, mass moment vectors.

CASTR' 2019, September 26-29, Siedlce, Poland


Acceleration of the second order – jerk of a rigid body rotates around a fixed point (II Second part)


APPENDIX 1.


Acceleration of the second order (Jerk) of a kinetic point moving with constant sectorial velocity as well as of the planet moving (I First part)
Katica R. (Stevanović) Hedrih


SALA WYKŁADOWA


SALA WYKŁADOWA